■□■ フーリエ級数展開のまとめ ■□■

		フーリエ級数展開		複素フーリエ級数展開	
		フーリエ級数	フーリエ係数	複素フーリエ級数	複素フーリエ係数
周期 2π		$f(x) \sim \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$	$a_k = \dfrac{1}{\pi} \displaystyle\int_{-\pi}^{\pi} f(x) \cos kx \, dx,$ $b_k = \dfrac{1}{\pi} \displaystyle\int_{-\pi}^{\pi} f(x) \sin kx \, dx$	$f(x) \sim \displaystyle\sum_{n=-\infty}^{+\infty} c_n \, e^{inx}$	$c_k = \dfrac{1}{2\pi} \displaystyle\int_{-\pi}^{\pi} f(x) \, e^{-ikx} dx$
	偶関数	$f(x) \sim \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx$	$a_k = \dfrac{2}{\pi} \displaystyle\int_{0}^{\pi} f(x) \cos kx \, dx$		$c_k = \dfrac{1}{2\pi} \displaystyle\int_{0}^{\pi} f(x) \left(e^{ikx} + e^{-ikx} \right) dx$
	奇関数	$f(x) \sim \displaystyle\sum_{n=1}^{\infty} b_n \sin nx$	$b_k = \dfrac{2}{\pi} \displaystyle\int_{0}^{\pi} f(x) \sin kx \, dx$		$c_k = -\dfrac{1}{2\pi} \displaystyle\int_{0}^{\pi} f(x) \left(e^{ikx} - e^{-ikx} \right) dx$
周期 $2L$		$f(x) \sim \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} \left(a_n \cos \dfrac{n\pi}{L} x + b_n \sin \dfrac{n\pi}{L} x \right)$	$a_k = \dfrac{1}{L} \displaystyle\int_{-L}^{L} f(x) \cos \dfrac{k\pi}{L} x \, dx,$ $b_k = \dfrac{1}{L} \displaystyle\int_{-L}^{L} f(x) \sin \dfrac{k\pi}{L} x \, dx$	$f(x) \sim \displaystyle\sum_{n=-\infty}^{+\infty} c_n \, e^{i\frac{n\pi}{L}x}$	$c_k = \dfrac{1}{2L} \displaystyle\int_{-L}^{L} f(x) \, e^{-i\frac{k\pi}{L}x} dx$
	偶関数	$f(x) \sim \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos \dfrac{n\pi}{L} x$	$a_k = \dfrac{2}{L} \displaystyle\int_{0}^{L} f(x) \cos \dfrac{k\pi}{L} x \, dx$		$c_k = \dfrac{1}{2L} \displaystyle\int_{0}^{L} f(x) \left(e^{i\frac{k\pi}{L}x} + e^{-i\frac{k\pi}{L}x} \right) dx$
	奇関数	$f(x) \sim \displaystyle\sum_{n=1}^{\infty} b_n \sin \dfrac{n\pi}{L} x$	$b_k = \dfrac{2}{L} \displaystyle\int_{0}^{L} f(x) \sin \dfrac{k\pi}{L} x \, dx$		$c_k = -\dfrac{1}{2L} \displaystyle\int_{0}^{L} f(x) \left(e^{i\frac{k\pi}{L}x} - e^{-i\frac{k\pi}{L}x} \right) dx$

$$\begin{cases} e^{i\theta} = \cos\theta + i\sin\theta, \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{cases} \iff \begin{cases} \cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}, \\ \sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i} \end{cases}$$

$$\begin{cases} c_n = \dfrac{a_n - ib_n}{2}, \\ c_{-n} = \overline{c_n} = \dfrac{a_n + ib_n}{2} \end{cases}$$