■□■ 離散フーリエ変換と離散フーリエ積分のまとめ ■□■

	離散フーリエ積分	離散フーリエ変換
数値列	$x_n = \frac{1}{T_0}\sum_{k=0}^{N-1} X_k {W'_N}^{nk}$ $(n = 0, 1, 2, \cdots, N-1)$	$X_k = \frac{T_0}{N}\sum_{n=0}^{N-1} x_n {W_N}^{nk}$ $(k = 0, 1, 2, \cdots, N-1)$
関係式	$X_k = X(k\Delta f)$	$x_n = x(n\Delta t)$
	$W'_N = e^{i\frac{2\pi}{N}}$	$W_N = e^{-i\frac{2\pi}{N}}$
	$\Delta t = \frac{T_0}{N} = \frac{1}{f_s}$	$\Delta f = \frac{f_s}{N} = \frac{1}{T_0}$
	$T_0 = N\Delta t = \frac{1}{\Delta f}$	$f_s = N\Delta f = \frac{1}{\Delta t}$

離散フーリエ変換 X_k の周波数スペクトル密度 $X_{T_0}(k\Delta f)$ は、以下の数式によって与えられる。

$$X_{T_0}(k\Delta f) = \frac{X_k}{T_0} = \frac{1}{N}\sum_{n=0}^{N-1} x_n {W_N}^{nk} \qquad (k = 0, 1, 2, \cdots, N-1)$$

$N = 4$ の場合：

$$\frac{1}{T_0}\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} {W_4}^0 & {W_4}^0 & {W_4}^0 & {W_4}^0 \\ {W_4}^0 & {W_4}^1 & {W_4}^2 & {W_4}^3 \\ {W_4}^0 & {W_4}^2 & {W_4}^0 & {W_4}^2 \\ {W_4}^0 & {W_4}^3 & {W_4}^2 & {W_4}^1 \end{pmatrix}\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

周波数領域では、以下の関係が成り立つ。

$$\begin{aligned} &\{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(3\Delta f),\ \cdots,\ X((N/2)\Delta f), \\ &\quad X((N/2+1)\Delta f),\ \cdots,\ X((N-2)\Delta f),\ X((N-1)\Delta f)\ \} \\ &= \{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(3\Delta f),\ \cdots,\ X((N/2)\Delta f), \\ &\quad X((-N/2+1)\Delta f),\ \cdots,\ X(-2\Delta f)\ ,\ X(-\Delta f)\ \} \end{aligned}$$

$N = 4$ の場合：

$$\begin{aligned} &\{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(3\Delta f)\ \} \\ &= \{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(-\Delta f)\ \} \end{aligned}$$

$N = 8$ の場合：

$$\begin{aligned} &\{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(3\Delta f),\ X(4\Delta f),\ X(5\Delta f),\ X(6\Delta f),\ X(7\Delta f)\ \} \\ &= \{\ X(0),\ X(\Delta f),\ X(2\Delta f),\ X(3\Delta f),\ X(4\Delta f),\ X(-3\Delta f),\ X(-2\Delta f),\ X(-\Delta f)\ \} \end{aligned}$$